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B.Sc. Part-I (Hons) Paper-II.

Successive Differentiation (Continue)

⑤ Find nth derivatives of $y = e^{ax}\sin(ax+b)$

Sol. Let $y = e^{ax}\sin(bx+c)$

$$y_1 = ae^{ax}\sin(bx+c) + e^{ax}\, b\cos(bx+c)$$

$$= e^{ax}[a\sin(bx+c) + b\cos(bx+c)]$$

$$= e^{ax}[r\cos\alpha \sin(bx+c) + r\sin\alpha\cos(bx+c)]$$

putting $a = r\cos\alpha$, $b = r\sin\alpha$

$$= e^{ax} r\sin(bx+c+\alpha)$$

Similarly, $y_2 = e^{ax} r^2 \sin(bx+c+2\alpha)$

$$\vdots$$

$$y_n = e^{ax} r^n \sin(bx+c+n\alpha)$$

where $r^2 = a^2+b^2$ and $\tan\alpha = \frac{b}{a}$

$$\therefore y_n = e^{ax}(a^2+b^2)^{n/2}\sin\left(bx+c+n\tan^{-1}\frac{b}{a}\right)$$

Similarly,

If $y = e^{ax}\cos(ax+b)$

$$y_n = e^{ax} r^n \cos(bx+c+n\alpha)$$

$$= e^{ax}(a^2+b^2)^{n/2}\cos\left(bx+c+n\tan^{-1}\frac{b}{a}\right)$$

Some nth derivatives of various type are---

Ex. ① Find the nth derivative of $\frac{1}{1-5x+6x^2}$

Soln:- let $y = \frac{1}{1-5x+6x^2} = \frac{1}{(2x-1)(3x-1)}$

$$= \frac{2}{2x-1} - \frac{3}{3x-1} = \frac{3}{1-3x} - \frac{2}{1-2x}$$

$$\therefore y_n = \frac{3(-3)^n(-1)^n n!}{(1-3x)^{n+1}} - \frac{2(-2)^n(-1)^n n!}{(1-2x)^{n+1}}$$

$$= (-1)^{n+1} n! \left[\left(\frac{3}{1-3x}\right)^{n+1} - \left(\frac{2}{1-2x}\right)^{n+1}\right]$$

Ex. ② Find the nth derivative of $\sin 6x \cos 4x$

Soln:- let $y = \sin 6x \cos 4x = \frac{1}{2}(\sin 10x + \sin 2x)$

$$\therefore y_n = \frac{1}{2}\left[10^n \sin\left(10x + \frac{n\pi}{2}\right) + 2^n \sin\left(2x + \frac{n\pi}{2}\right)\right]$$

Ex. ③ Find nth derivative of $\sin^2 x \cos^3 x$

Soln:- let $y = \sin^2 x \cos^3 x = \sin^2 x \cdot \cos^2 x \cdot \cos x$

$$= \left(\frac{2\sin x \cdot \cos x}{2}\right)^2 \cdot \cos x = \frac{1}{4}\sin^2 2x \cos x$$

$$= \frac{1}{8}(1-\cos 4x)\cos x = \frac{1}{8}\cos x - \frac{1}{8}\cos 4x \cdot \cos x$$

$$= \frac{1}{8}\cos x - \frac{1}{16}(\cos 3x + \cos 5x)$$

$$= \frac{1}{16}[2\cos x - \cos 3x - \cos 5x]$$

$$\therefore y_n = \frac{1}{16}\left[2\cos\left(x+\frac{n\pi}{2}\right) - 3^n \cos\left(3x+\frac{n\pi}{2}\right) - 5^n \cos\left(5x+\frac{n\pi}{2}\right)\right]$$

Ex. ④ If $y = x + \tan x$, show that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$

Soln. $y = x + \tan x \Rightarrow \frac{dy}{dx} = 1 + \sec^2 x$

$$\frac{d^2y}{dx^2} = 2\sec x \cdot (\sec x \cdot \tan x) = 2\sec^2 x \cdot \tan x$$

$$\text{L.H.S} = \cos^2 x \cdot 2\sec^2 x \cdot \tan x - 2(x + \tan x) + 2x$$

$$= 2\tan x - 2x - 2\tan x + 2x$$

$$= 0 = \text{R.H.S.} \quad ; \text{proved.}$$